



Hints on how to improve mathematics instruction

Flavia Colonna¹, Glenn Easley²

¹ Department of Mathematical Sciences, George Mason University (Fairfax, Virginia, U.S.A.) - fcolonna@gmu.edu

² System Planning Corporation (Arlington, Virginia, U.S.A.) - geasley@sysplan.com

Abstract

Teaching mathematics to prepare effectively students toward the achievement of their potential requires an understanding of what helps them learn and the adoption of innovative tools that exploit the existing technology in the classroom. The need to account for students' abilities and weaknesses in designing a lecture and inspire them is at the basis of a proper teaching strategy aimed at acquiring high competency in mathematics tailored to their needs and at recruiting students to scientific disciplines and in engineering. The adoption of mathematical software integrated with the lectures and the use of interactive teaching tools are important modifications needed to reach this objective.

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1 Introduction

Mathematics is an academic subject whose knowledge to some degree is required in most disciplines, and in particular, in the natural sciences and engineering. Unlike many scientific disciplines that have evolved over time, particularly as a consequence of scientific discoveries and technological innovations, the type of mathematics taught at the primary, secondary, and university education levels has not changed significantly in the last century. Aside from an increased early exposure to probability and statistics, or a taste of set theory even in elementary school, subjects such as algebra, geometry, or calculus, have remained virtually unchanged from one generation to the next. Yet, in recent times, the teaching of mathematics has been greatly analyzed and debated. How do we prepare students? How do we create the conditions to attract them to this subject? What can be done to close the gap in mathematics achievement between men and women? How can we help our students achieve the best performance possible?

Virtually everyone agrees that mathematics is important for research and development, for technological advances, and for preparing our students and help them become competitive men and women in the workplace. Economic growth depends on the creation of new technologies. After all, mathematics is classically a problem-solving discipline. Yet, in the United States it is common to admit ignorance in mathematics almost with a sense of resignation or pride. The popular culture is decidedly at odds with the vision of a mathematical whiz as 'cool'. Thus, it is necessary to change this popular culture; but how? The starting point is to thoroughly change the teaching methods that only work for a limited number of students. Of course, this change of mentality has to begin in elementary school and continue throughout secondary school. Indeed, by the time a student reaches the university, it is often too late to change bad habits and abolish preconceived notions about the subject.

In recent years the division of undergraduate education of the National Science Foundation (NSF) has been actively promoting research and awarding grants for course curriculum development or modification in science, technology, engineering and mathematics through a program called STEM. For details on this and related programs, see <http://www.nsf.gov/funding/>. The NSF publishes and regularly updates statistics on education, research, enrollment, and graduation rates for undergraduates and graduate students, also categorized by gender and minority status. For details, see <http://www.nsf.gov/statistics/>. These statistics document the increase in doctoral degrees awarded in the U.S. to foreign students, often out-ranking the U.S. citizens. This has raised significant concern, as many foreign graduates will return to their countries of origin and enrich them culturally and economically. Through powerful investment in

education, it is hoped to reverse this trend. In other countries, such as Italy, there is the opposite problem: the need to stop the ‘brain drain’ through legislation and financial incentives.

In this paper, we share some ideas on how to make changes in the curriculum to take steps toward the creation of a better learning environment. The main focus here will be on teaching mathematics at the university level, although most of what we indicate is applicable to most grades of secondary school. While these ideas are not novel, for the most part they remain confined to certain schools of thought. We hope this work will have the effect of informing educators about ongoing methodologies as well as establishing connections for discussion and exchange of materials on teaching mathematics.

2 Different models in higher education

There are two substantially different models for university systems throughout the world. One, such as the U.S. undergraduate education system, is the so called liberal arts model, which is based on teaching a wide variety of subjects with a specific set of requirements, known as general education requirements. These requirements have to be fulfilled by all students, regardless of the field in which they wish to specialize. Depending of the type of degree sought, B.A. (Bachelors of Arts) or B.S. (Bachelors of Science), for a student who seeks a degree in a specific discipline, the percent of courses related to that discipline could vary roughly between 40 and 70 (i.e. see requirements in the (George Mason University, 2010-2011)). The aim is the development of a cultural breadth. The other model, such as the Italian university system, is based on studying almost entirely courses related to the discipline of choice. In the latter model, the aim could be seen as an over-specialized professional education and focus. For this reason, the issues regarding teaching a subject such as mathematics in the two models vary significantly. For more information on the liberal arts model, we recommend to the reader the American Association of Colleges and Universities website: <<http://www.aacu.org/leap/>>.

2.1 The liberal arts model

The principle behind the liberal arts model is that cultural breadth provides a student with a more complete education at an age when he/she has already acquired a higher level of maturity. The in-depth specialization in one discipline takes place after graduation, if so desired, in the pursuit of a Masters or a doctoral degree. It is the belief of those who favor this form of education, as opposed to the specialized degree model, that students who graduate from a liberal-arts institution are more fit and prepared to enter the white-collar workforce.

The first issue that arises is: What constitutes a reasonable set of general education requirements in mathematics? And, most importantly: What skills should a student develop through the relative courses? In the U.S. model, a desirable skill is competency in analytical reasoning. Thus, a prospective general education course in mathematics has to teach how to think mathematically using methodologies based on logic. A course of this type does not require an extensive background in mathematics, does not involve memorization of formulas, and is not based on learning mathematical theories. A course in analytical reasoning, in fact, has a modest amount of mathematics. The aim is to teach how to think by starting with a concrete problem, interpreting it symbolically, and using tools such as the principle of mathematical induction or logical reasoning to derive a solution through a series of steps starting from known facts. The ability of an instructor is to disassemble the problem into parts in order to obtain as much information as possible, embed in the process hints of what strategies pay off and discuss the options with the students, seeking their opinions and raising their interest, while at the same time guarding against logical 'traps' or circular arguments. An effective teacher should also take the opportunity to explain why certain strategies do not pay off and offer alternative routes.

The development of analytical reasoning skills is enforced throughout the course by evaluating the students by means of exams and quizzes that contain problems they have not seen before. However, given the limited amount of time students have to work on test and quiz problems, most of the higher-level learning by a student takes place in the form of demanding take-home assignments for which students are given at least a week to complete.

Mathematics classes for non-mathematics and science majors are typically given in medium size classrooms to facilitate communication between student and teacher in the classroom. Since the objective is to teach how to solve problems through analytical reasoning, it is useful to supplement the standard course materials with applications from the real world. In fact, it is often the case that even in publications such as newspapers and magazines, statistical information is given erroneously or the article writers are not sufficiently knowledgeable in the topics they cover and end up making mistakes in their reporting. It is particularly important for a student to realize that, just because an article appeared in a reputable publication, one should not assume that its content is accurate without a proper analysis. When reporting, for instance, on a study done to determine the effectiveness of a drug or to establish a cause-and-effect link, such as between a high-fat diet and heart disease, one should not deem as trustworthy a study done on a small sample or choosing a non-random sample. Furthermore, there is so much uncontrolled advertising we are daily exposed to, claiming the benefits of a particular product on one's health

or fitness or purporting the superiority of a certain commodity. How do we discern facts from unproven claims? Students should therefore learn what to look for in their analysis of advertisements and reports, even those considered scientifically sound.

2.2 The specialized professional model

A model based on studying courses related to the field of interest should in theory have a more integrated picture of the role of any required discipline for learning that field. However, in practice there is often a disconnect between the teaching of the discipline and how it could be used effectively in the specialized field. The reasons for this dissociation are often due to poor communication between different departments and the belief that the subject at hand is needed in its ‘pure form’, without much thought on applications to make it more relevant to the students. In addition, instructors of the so called ‘service courses’ (such as many mathematics courses) often delegate the coverage of applications to the specialized field. However, the context in which an application is shown is different and, by the time the application is shown in the specialized field, the mathematical aspect are often long forgotten or no longer fully understood by the student. This delay has detrimental effects on retaining and seeing in action the mathematical techniques learned in the course.

It is our experience that even for students of disciplines that have a heavy mathematical content, such as engineering, there is a need for a more integrated course structure with plenty of examples from engineering where a particular mathematical tool is needed. An effective teacher shows how to deal with a particular application before and after learning a certain technique. Only then can a student fully appreciate the power of the method.

Another important component on learning in the pursuit of a specialized degree is to assign projects with a creative component by offering precise guidelines, but also a certain freedom to reach a certain practical objective. In fact, these assignments are often the best opportunities for meaningful learning to take place. Better yet is to assign open-handed problems, in which a student is asked to accomplish a certain task, possibly too difficult to be done independently, without much direction or initial guidance. However, to offset this push for autonomy it is important for the instructor to remain available for questions, clarifications and suggestions. In Section 6, we shall discuss more in detail the issues regarding the teaching of mathematics courses with an applied component to be implemented at the computer that we have taught.

3 Teaching philosophy

What constitutes the perfect teaching style? There cannot be definitive answers to this question since students have different style preferences based on their individual strengths. However, most of us would agree on certain parameters in evaluating teaching effectiveness. The lecture should be clear, stimulating, whenever possible even thought provoking, and rich in interaction between the teacher and the audience. The amount of detail presented in class should not be excessive and students should be asked to try filling in the holes of proofs that follow arguments students have already been exposed to or complete calculations on their own. The best way to learn a new subject is to make connections with other related topics already known, emphasize the key ideas of the new theory or technique and at the end of class summarize the main points raised in the course of the lecture. The repetition of some aspects covered at length previously helps a student sort out the important information from the rest. Not all students are capable to be fluent at synthesizing crucial elements of a concept or method. Ideally, each student should be able to write a short paragraph on what was learned that day.

A very important quality every teacher should possess is to inspire. A student cannot become excited about learning by being a passive spectator in a classroom governed by an instructor embedded in silence. For a subject such as mathematics, this is a particularly challenging issue. Aside from the logistics on how to create an environment amenable to a two-way communication between students and teacher, there is the issue of class size, the need for moving forward with the curriculum, and to involve in the discussion those students who prefer to stay by the sidelines. Thus, while this interactive learning environment is not practical in a class with over 35 students, there are other strategies that could be implemented as a way to increase the communication at least outside the classroom.

In the mid 1990s one of us participated in a year-long series of workshops on the pedagogy of math and science. In the course of the workshop, there was a debate on how to best increase students' participation, test their mastery of the subject and give them the tools to learn at a deeper level. A comparison among different lecture styles for different cultures was shown. Videos showing a typical class were analyzed. The videos from a non-English speaking class had subtitles to help the viewer understand the dialogs. The workshop's participants then discussed the positives and the negatives of the different styles.

One of the problems in a class of geometry in a high school in the United States was the excessive repetition of facts that did not go beyond learning the name of a geometric entity. There was no discussion on the purpose of the con-

cept being taught, or how this concept could be implemented. While the class took place in a lively classroom with an informal and relaxed atmosphere, the students were distracted, bored, and did not pay attention to the teacher.

By contrast, in a Japanese classroom, in which there was a much more formal atmosphere, the teacher started the class with a practical problem on how to create a fence dividing the lands of two neighbors according to certain criteria. He sought the students' opinions and the discussion lasted for a long time. Everyone participated in the discussion and only at the end the teacher developed the concept in full. By then the students had already learned the key ideas and this made the lecture very easy to understand. At the end the students were divided into groups to explore new problems related to the topic developed. There was nothing repetitive about the approach adopted by this teacher and the students seemed genuinely interested.

This workshop made an impression on the participants who then wondered why no university faculty is required to be trained in teaching methods. Instructors are certainly affected by their teaching experiences in the course of their academic careers, yet, often they tend to emulate their own professors without giving much thought on whether there are better teaching alternatives, thus perpetuating a model that has the potential to be greatly improved. This is precisely why an exchange program that allows faculty to experience teaching in different settings can bring the breath of fresh air needed to make classroom improvements.

4 The gender gap

Statistics in the United States indicate that the number of women who choose to specialize in fields such as mathematics, computer science and engineering is quite small and this gender gap persists in their career choices after graduation. The motivation for this gap has been attributed to women's lower achievement in mathematics and science in secondary school and at the university than that of men. This realization has been a key factor in trying to promote more awareness of the necessity to increase interest in mathematics among girls in the early grades, since girls' negative attitude toward mathematics has the effect of turning them away from many scientific disciplines. Initiatives at giving priority to women for admission at selective colleges and universities in disciplines in which they are underrepresented have been hotly debated. Aside from the deserving male applicant who complains of having been unfairly turned down to make room for a less qualified female applicant, there is a significant concern that the quality of education may suffer as a consequence of this policy.

In (Else-Quest *et al.*, 2010), Else-Quest, Hyde and Linn analyzed cross-na-

tional patterns of gender differences in mathematics. This work was motivated by the continuing concern on the impact that gender differences in mathematics achievement and attitudes has on the underrepresentation of women at the highest levels of science, technology, mathematics, and engineering, and by the persisting stereotypes according to which girls and women lack mathematical ability, despite growing evidence of similarities in math achievement between genders, as documented in several recent studies such as (Hapern *et al.*, 2007). In their study, in which two large international data sets were analyzed, in (Else-Quest *et al.*, 2010) the authors showed that a gender gap in mathematics achievement persists in some nations but not in others, and that on the average, males and females differ very little in math achievement, despite the more positive math attitudes among males. The differences are more pronounced at the top levels of performance. However, their findings were not consistent with the theory supported by several researchers (e.g. (Baker & Jones, 1993)) suggesting that cross-national patterns of gender differences in mathematics achievement are related to gender inequities in educational and economic opportunities present in a culture. They argue that factors that have a more direct effect on learning, such as the curriculum and the quality of instruction, may be more useful to lessen the gender inequity on mathematics achievement.

Another theory on a factor influencing the gender gap is the stereotype threat experienced by members of a gender, race, class or status when they believe that they might be treated negatively simply because of their social identity. According to this hypothesis, girls perform less well in mathematics because they are stigmatized as a result of their gender identification. Consequently, women who are stigmatized in fields with a heavy mathematical component will tend to avoid them. For more information on this theory, see (Murphy *et al.*, 2007) and (Rydell *et al.*, 2010).

One of us has been involved for several years in the outreach program 'Blueprints to the future' organized in the 1990s by the American Association of University Women. This program recruited participants among several middle school students in Fairfax County (Virginia, U.S.A.) to participate to a math awareness day in which many well-known speakers in different scientific fields were invited to talk about the nature of their work and the students were invited to take part in hands-on activities. The emphasis of this program was on sending out the message on the importance of mathematics in all fields presented in the program.

Other initiatives aimed at reducing the gender gap are individual mentoring of girls and the active recruitment of female students through advertisements and public speeches by academics and researchers from the private sector through school visits.

An interesting realization confirming the influences of the environment

on math achievement can be seen by the effect of video game playing. Girls can improve performance in mathematics and science by playing specialized video games designed to improve visual processing. Indeed, there is ample literature supporting the hypothesis that playing video games helps the player strengthen the hand-eye coordination as well as visual and spatial abilities. In 2009, U.S. president Barack Obama launched a campaign called Educate to Innovate whose scope is to improve the mathematical, scientific, technological, and engineering abilities of American students by harnessing the power of interactive games.

5 Different learning styles

An effective teacher must have the ability to gear his/her lectures to a diverse audience. Thus, the style that is best suited to have a strong impact on learning is one that relies on multiple techniques: visual, for those students who have difficulty with verbal instruction, auditory or through the adoption of manipulatives, for those who do not interpret geometric features properly or have less developed spatial abilities, and illustrative examples for those students who have poor abstract reasoning skills. In addition, instructors must keep in mind that an increasing number of students has limited proficiency in the language spoken in the classroom or learned differently how to perform arithmetic operations, which may have a negative impact on his/her ability to follow a calculation or the mathematical steps of a proof.

Special-needs students, such as dyslexics, students affected by attention-deficit disorder or other forms of impairment deserve a particular care by the instructors. It is therefore important to establish contacts outside the classroom to understand what learning strategies may help them and create lesson plans accordingly, or develop supplemental materials to hand out to the students who need them.

A helpful teaching tool is to provide students several days after the lecture with a copy of the instructor's own lecture notes, preferably formally typed. A common problem that arises is that a student who is engaged in taking notes may lose touch with the instructor's verbal explanations, which prevents the lecture from being processed in a coherent and complete manner. The preemptive distribution of lecture notes may have the undesirable effect of discouraging students' participation since they could feel not motivated to attend. We believe that attendance and class participation are important factors for optimal learning. It is important for the students to write down as much of the lecture as possible in order to retain most of the information to be recalled afterwards. It would lessen the students' anxiety the knowledge that they can have access to a written copy or an electronic version of the lecture at a later time provided that

they continue attending regularly. Of course, this strategy is time-consuming and may not be feasible or practical for many lower level courses.

For an excellent resource on teaching strategies for students with different abilities, we recommend to the reader (Armstrong, 1994; Gardner, 1993).

6 Technology in the classroom

Many classes at universities across the U.S. are being held in ‘smart classrooms,’ that is classrooms equipped with a multimedia console hooked to a computer and connected to a DVD player, a projector, and a lit writing board that can be projected onto a large screen. An instructor can then have instantaneous access to lectures online as well as demonstrations of math software with plotting and computing capabilities. There are many advantages in the use of these technological tools. The screen is better lit and more visible from the back of the room than the traditional blackboard. Instructors can easily access old materials and more steps can be shown using color coding and other visual devices aimed at emphasizing certain aspects of the lecture. These tools can also be used to project onto the screen pictures, formulas or problems from a textbook or another source. Aside from the occasional computer glitch or equipment failure, these classrooms have revolutionized the ways many subjects are taught and have opened up many new possibilities on how to upgrade classroom instruction, both in terms of content and visually.

An optimal environment for learning math courses such as calculus or differential equations is based on the adoption of software whose aim is to obtain a quick answer to a computation with the push of a button or graphing instantly a function for demonstration purposes. The types of math software we are most familiar with are Maple and Matlab.

Maple is suitable for a wide variety of uses, ranging from solving an equation numerically, to differentiating and finding limits of a function, as well as plotting the graph of a curve in polar or parametric form. This software is also necessary in order to compute definite integrals of functions whose antiderivatives cannot be determined analytically and sums of convergent series. There are commands that allow for an increased resolution to yield a more precise plot, by far superior to what is feasible with a hand graphing calculator.

Matlab, which is heavily used in engineering applications, can also be utilized for graphing, but its main purpose is to carry out matrix operations or to implement codes based on the use of linear algebra.

These tools are very useful also to explain the meaning of a linear approximation of a function. By zooming in on a suitably small interval or grid and plotting simultaneously the function and its linear approximation, it is possible to demonstrate the accuracy of the approximation and even determine the size

of the error before a mathematical estimate of the error could be determined analytically or through a formula. A follow-up problem that can be tackled next is to see how a quadratic approximation, or more generally, a polynomial approximation can be implemented for better results and how the degree of the polynomial should be chosen to obtain the desired level of accuracy with the smallest number of operations.

The intent of changing the calculus curriculum to one aimed at making decisions on modeling, or approaching a problem through the use of technology is to make students more aware of the methodologies taught by first giving them a hands-on experience. The aim is also to teach students how to apply these tools by focusing less on programming and more on how to choose the technology that best fits their needs.

A problem that arises in teaching specialized mathematics courses that are applied in nature, such as wavelet theory and Fourier analysis, is that these courses are typically structured by exposing the students mostly to the theoretical background and leaving very little room to the applications. Such courses offer unique educational challenges, not only for the difficulty integrating the pure and the applied side of the subject, but also because the type of mathematics needed in the application is often different, leaving a gap between the two to be filled. Indeed, going back to the examples mentioned above, both wavelet theory and Fourier analysis have a rich theoretical background in analysis and can be taught by just focusing on their classical theory. Teaching these courses by following such an approach is what is often done because this is how the instructors themselves have learned these topics and is the way in which the material is presented in most textbooks. Yet, such instructions can miss an important connection with applications. An important value of these subjects is indeed in their applications. Ironically, a student may successfully complete such courses without being able to do any real world applications.

An important issue is that the most useful applications of these subjects are done by using computers. When the focus of these courses is too theoretical, no basic instruction is given on how to implement the concepts covered on a computer. Not only does this create an obstacle for helping students develop new technology using such theories, but it can also be misleading. Indeed, often the applications in the digital domain become finite and discrete, and hence associated to linear algebra rather than analysis. This implies that many of the theoretical results should also be formulated according to the language of linear algebra. Yet, doing this adaptation, in addition to presenting the analysis results, takes away much of the available time in the lectures, leaving little time left to present the related algorithms. Thus, a useful technique is to design assignments that gear the students toward developing some of this knowledge by themselves. However, in order for this to work, the students still need to see

many basic demonstrations in the classroom.

It is our experience that these demonstrations seem to work best when presented as combinations of improvising and following prepared scripts, which should be provided to the students at a later time. It should be noted that improvising with a computer software package such as Matlab is most helpful at creating a friendly student-teacher interaction. Part of this happens because the instructor will often need to sit and type some commands creating a less imposing posture and an atmosphere more conducive to having an open dialog. In addition, the improvisation can be enjoyable, exposes the students to what is like to do code development, and give them an impression of what active researchers do. This is important because the classical theory, while fundamental, can often leave the students wondering what could possibly be done that is new to the field.

7 Technology and the publishing industry

The impact that technology can have on teaching has been understood for some time and the publishing industry has been quick at taking advantage of the emerging technologies and at marketing new textbooks with a large number of calculator or computer applications. Most well established and popular textbooks in lower level mathematics courses have been replaced by updated versions by the same authors with increasing frequency, due especially to how quickly technologies are evolving. It is not uncommon to have a new edition in the market only two or three years after the preceding one has appeared. In addition, a new generation of technology-rich textbooks and user-friendly tools has emerged.

New learning tools that are gaining in popularity are complete online courses affiliated with textbooks by a specific publisher. The instructor can create online courses for as many textbooks as desired and can choose whether or not to make the course available to the students. The instructor can also set aside practice problems with solutions, multiple choice quizzes and tests for credit and allow students to access other material. One of us is familiar with the interactive website called MyMathLab, which, working jointly with Pearson's Education Publishing Company, provides self-test and works through practice exercises with step-by-step help. MyMathLab includes multimedia learning aids, videos, animation, as well as live tutorial help. For more information, see <http://www.mymathlab.com> and <http://www.pearsonhighered.com>. Another interactive website that offers course development services to help create engaging courses to fit the instructor's needs is Cengage Learning (for information, see <http://www.cengage.com/custom/>). This type of online course set up, however, is not sufficient for most students to be a satisfactory substitute of the standard

in-classroom learning environment. Indeed, several of our students who chose to use this online material requested help from one of us because they could not understand the solutions provided or why a certain answer they gave was incorrect. In order to provide this service to the students, the instructor has to monitor the choice of materials and testing and be available for online support when requested.

We believe that, as it is already happening in other fields, even for mathematics a combination of in-class (according to the criteria described above) and interactive online course set up is ideal for providing clarity and support and for assessing effectively what the student has learned. Moreover, it has the potential to become the prototype of a self-paced course for students who cannot come to class regularly.

Conclusions

In this article, the most important aspects on teaching mathematics have been outlined. The main objective of this work was to promote discussion and expand the study on how to foster a better learning environment for mathematics at the university. It is our hope that this article will stimulate an exchange of ideas on pedagogy and the creation of new tools for learning both inside and outside the classroom, as well as emphasize the importance of paying particular attention to girls' learning and of adapting the teaching style to students with different learning abilities.

In the United States there are several federal agencies and academic mathematical organizations (most notably, the National Science Foundation, the American Mathematical Society, the Mathematical Association of America, and the Association of Women in Mathematics) whose mission is to facilitate the exchange of information among academics and researchers. These organizations have been instrumental in the cultural and social awareness raised in their newsletters and publications, and through conferences. There are many opportunities to expand this exchange by establishing new networks through blogs and other means of communication. A first step is to establish cross-cultural connections and assess periodically the progress being made socially as well as in the teaching and research arenas.

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