

Technology to enable new paradigms of teaching/learning in mathematics: the digital interactive storytelling case

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Abstract

This paper concerns the design and implementation of a particular methodology for mathematics teaching/learning which exploits an interactive and immersive metaphor of storytelling. This research aims to promote processes such as inquiring, conjecturing, formalizing, proving in mathematics, and to investigate which is the best way to organize ICT tools to achieve that purpose. We also report the findings of an ongoing experimentation at the K12 school level.

KEYWORDS: Digital Storytelling, Mathematics Education, Collaborative Learning, Moodle.

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- to amplify the reach of peer-tutoring, providing more efficient tools to support students with difficulties;
- to use suitable tools to perform Social Network Analysis to evaluate the organization in cohesive aggregation by means of algorithms for detecting communities (Polo, Dello Iacono, Fiorentino & Pierri, 2019).

1. Introduction

This study concerns the PRIN project “Digital Interactive Storytelling in Mathematics: a competence-based social approach”, aimed to define a socio-constructivist methodology to build didactic activities within a competence-oriented mathematics education framework, named DIST-M (Digital Interactive Storytelling in Mathematics) (Albano & Dello Iacono, 2018). More specifically, the objectives of the project are:

- to improve the DIST-M methodology by means of more adaptive collaborative scripts, able to better support social interaction within online group(s) and useful for more personalized path, with respect to the roles that students play during the tasks;

The approach is based on the use of collaborative scripts (King, 2007; Kobb et al., 2007) within a sequence of Vygotskian tasks (Vygotsky, 1980) embedded in a digital storytelling framework. The methodology provides for interaction among peers on an e-learning platform (Moodle) (Albano, Dello Iacono & Fiorentino, 2016) integrated in a suitable way with other kinds of semiotic mediators, especially suitable for mathematics learning.

This paper is focused on the design and implementation aspects of DIST-M, paying attention to the role of (inter)mediation of the platform. We investigate how to exploit a platform to design learning activities embedded in storytelling context aimed to support the development of students’ argumentative capabilities in mathematics.

Here we present the theoretical framework and the starting point for the implementation of the collaborative

scripts. Then we move to the technological features. Finally, we draw some conclusions from the experimentations in the classes.

2. Theoretical framework

We refer mainly to two theoretical frameworks in mathematics education literature: the discursive approach to the teaching and learning of mathematics (Sfard, 2001) and the story problems (Zan, 2012).

According to Sfard, thinking is a case of communication and languages are not only carriers of pre-existing meanings, but they are constructors of the meanings themselves. Learning becomes the participation to a discourse, that is the “mathematical discourse”. In our case, the discourse takes place in a mainly written communication, since it develops in digital platforms. This could appear as a constraint, instead it should be assumed as a strong point. Indeed, in accordance with Radford (2002), writing is considered as a semiotic tool for objectification, used from individuals in the social processes of production of meanings, to gain a stable kind of awareness, to make explicit and visible own thoughts and to accomplish actions.

The story problems are those in which the mathematical structure is embedded into a familiar situation for the student and takes the shape of a narrative. To understand the story, the student has to resort not only to logical thinking, which is the one that deals with explaining what happens in a deductive logic, but also to narrative thinking, which is the one making sense of things, dealing with intentions, desires, beliefs and feelings (Bruner, 1986).

In order to allow the story to support the problem-solving process, the narrative and logical dimensions should be well combined (Zan, 2012). Zan emphasizes that the issue should naturally arise from the need to achieve a purpose and the student should imagine that his resolution activity can influence the story.

The collaborative and Vygotskian approach, based on social and individual construction of knowledge, supports the development of argumentative and communicative skills (Lazarou, Sutherland & Erduran, 2016). Students, engaged in the activities planned by the scripts, analyse and explain their reasonings, conjecturing arguing and interacting with their classmates in order to persuade them of the validity of their argumentations and to take into account those of the others. In order to improve the collaborative learning experience, we adapted the scripts to individual and group characteristics, carrying out adaptive collaboration scripts (Baker, 2003), which are very effective in promoting a better self-regulation of learning (Demetriadis & Karakostas, 2008), especially in online environments (Azevedo et al., 2005).

3. The case study

From a mathematical point of view, DIST-M aims to support the development of students’ abilities in algebraic modelling, in producing conjectures, argumentations and proofs. The mathematical task that students face is:

“Choose four consecutive natural numbers, multiply the two intermediate numbers, multiply the two extremes, and subtract the results. What do you get?” (Mellone & Tortora, 2015).

The problem is embedded in a story and given in a narrative way (Liljedahl, & Zazkis 2009; Zan, 2012).

As the narrative progresses, the story evolves according to the characters’ interaction with it. The genre chosen was science fiction, which sees a group of four friends engaged in the task of communicating with aliens from whom they had received mysterious messages made up of numbers and operations.

Four friends correspond within the story to four characters following described:

- *Marco*, the BOSS, leader of his group of friends, and has obtained with his ways of doing, the trust of his companions.
- *Sofia*, reading lover, enjoys writing, wants to be a journalist and has the obsession of the blog. In fact, they call her the BLOGGER.
- *Clara* has a diffident personality and often torments her companions with doubts and questions...she's a real PEST.
- *Federico*, computer lover, is convinced of the existence of extra-terrestrial and he is always looking for creative and brilliant ideas to share with friends; for this reason, he is named PROMOTER.

The four students of the group will agree among themselves within Chat tool their role in the story, considering the aspects of their personality.

Alongside the four friends, there is also an adult, Gianmaria, Federico’s uncle, also an expert in computer devices and a lover of mathematics. Gianmaria (the GURU in the story) is the teacher/tutor’s avatar and acts as an expert in the learning process.

The story problem starts from Federico who, fascinated by life beyond the planet Earth, has developed an electronic device in the hope of capturing some signal from the space. And finally, one day, a sequence of characters appears on the screen.

Unfortunately, during reception the device breaks down and the message remains incomplete. Federico’s curiosity is too strong that he involves his friends in this adventure, looking for the meaning of those signs and a way to communicate with the aliens.

To get help in the task, the four friends decide to involve Gianmaria, who agrees to help them solve the enigma.

The interactions among them and with Gianmaria guide the flow of the story (and of the learning path), and lead them to continue communicating with the aliens, after Federico has fixed the device and also connected it with the smartphones of each of them.

This new feature allows aliens to send a different message to each of them, so that in a second phase the group is called to think about different sequences and find out what they have in common (in the case of consecutive odd and even numbers, which lead to 8).

The story has been divided into episodes, so you can add or choose to enjoy an indefinite number of episodes. Each episode gives space for new questions.

4. Design of DIST-M

All the learning activities take place in the context of a narrative, an engaging and familiar situation for the student. The genre chosen was that of science fiction, which sees a group of four friends working in the enterprise of communicating with aliens from whom they had received mysterious messages made up of numbers and arithmetic operations. In addition to the four friends, there is also an adult, who is the teacher/tutor's avatar and acts as an expert in the learning process. Each student is a character of the story (Liljedahl, 2007; Albano, Pierri & Polo, 2019) and the personal and group interactions are moderated by the teacher through the character assumed in the story. The story evolves over time following the characters' interactions with it.

The whole activity of DIST-M consists of different tasks, as shown in Figure 1.

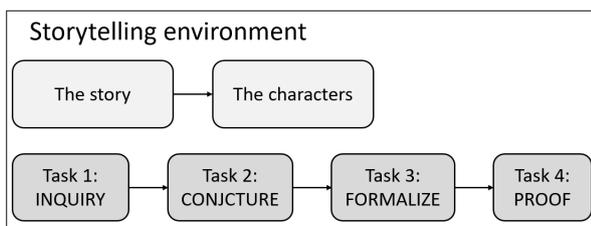


Figure 1 - The tasks.

The students are embedded in a storytelling environment, consisting in comics strips. The story and its characters are firstly introduced. Then the students are engaged in four tasks, which develop according to the story, as described below:

- *Task 1 - Inquiry*: the students, starting from their observations, produce a description of what has been observed.
- *Task 2 - Conjecture*: the description obtained is rearranged to produce conjecture in verbal language.
- *Task 3 - Formalize*: the students formalize the conjecture produced to prepare the way for the proof.

- *Task 4 - Proof*: here the students must construct the proof and to justify each step of the deduction.

Students are divided into Characters and Onlookers. Characters assume the roles of Boss, Blogger, Pest, and Promoter (see Section 3). In each episode, the group of Characters plays as the protagonist while the Onlookers, also divided into groups, observe the Characters during the carrying out of the task. Each onlooker observes a specific role. Roles, both Characters and Onlookers, are exchanged in each episode, so that each student plays a different role, as a character or as an onlooker, to internalize them (Vygotsky, 1980). Each task provides for some moments of individual work and some others of collaborative work and discussion with the expert. The expert acts as mediator intervening as necessary in informal moments of work among peers (using the chat) and in moments explicitly reserved to the debate (in the forum). Moreover, the expert has a favourite communication channel with the Promoter, to start the problem-solving phase or if in impasse. Her role of mediator regards both the mathematics and the communication.

5. Implementation and role of technology

The theoretical framework described in the previous sections guided the design of the DIST-M and its implementation for a case study. In this section we provide a brief description of how this was done using an online learning environment.

The implementation of complex learning activities as the those described so far requires a sophisticated and flexible environment like Moodle. In fact, all activities and (social) interactions planned in the DIST-M were implemented with Moodle, choosing and fine-tuning the most suitable tool for each didactic and communicative task.

The implementation considered all the main aspects of the design: the narrative setting, the small group approach, the importance of communications as a way to foster argumentative skills, the metacognitive reflection.

We choose the best Moodle tool for each communication activity according to the expected linguistic register. So, for instance, we used Chats to handle all immediate and informal discussions between the characters and for private messaging between the Promoter and the Guru. On the contrary, we used Forums to manage all formal discussions between characters and the Guru, promoting the transition towards more literate registers (Ferrari, 2004).

We used Moodle groups to partition students into teams of 5 member/characters: Boss, Promoter, Blogger and two Pests. Each group was isolated from the others using Moodle's separated group mode (which gives the illusion of a separate course for each group, virtualizing the activities of one course), this allows students to work

in small groups while granting the Guru (with a suitably configured Moodle role) the privileges to monitor all activities and step in when invoked or necessary.

A specially crafted Moodle role was used to implement the onlookers, which can see all discussions between characters in Chats and Forums but are not authorized to participate. Moreover, onlookers belonging to the same group can communicate with each other by means of dedicated Chats whose visibility and access permission is restricted by checking their special role.

The narrative approach has been fully supported by some customizations that allowed us to design the entire user experience of the student as if he were inside a comic book. In fact, each episode of the story is implemented with a Moodle Book resource whose pages are filled with using comics created with the online environment Toondoo and Microsoft PowerPoint. Furthermore, the whole learning environment looks like a comic strip, thanks to some labels/comics used as links to stealth activities (available but hidden in the homepage of the course). A few lines of custom CSS allowed to adapt to this immediate and unconventional layout almost all the other activities and resources.

Several parallel didactic paths were built, foreseen by the instructional design described in the previous sections, have been enforced using Moodle's access conditions, leveraging on the belonging to the various groups/characters of the story. For example, the private chat between Promoter and Guru is visible only to the former; the Questionnaire simulating the emailing is only visible to Bloggers; the dedicated Onlookers Chat is visible only to them.

Finally, many Moodle tools have been expanded integrating or embedding digital applications such as interactive GeoGebra constructions Albano & Dello Iacono, 2019a). Their availability, however, can be chosen by the Guru, according to the needs and mood of the discussion.

More in detail, three GeoGebra applications have been created and embedded within Moodle activities:

- a (GeoGebra) spreadsheet (view) is embedded to support the students during the inquiry (task 1); it allows to play with the quadruples, insert new ones, and quickly test relationships on all of them (Figure 2);
- two Interactive Semi-open Questions (ISQ) (Albano & Dello Iacono, 2019b) to support the conjecture production (task 2) and the formalization (task 3) respectively. These that allow the assembly of mathematical statements (sentences) by dragging digital tiles (words) (Figure 3). We chose the digital tiles in such a way as to make available to the students the main keywords, numbers, letters, causal conjunctions, symbols, and mathematical expressions. The aim is to allow the student to produce verbal conjectures and statements, in a formalized mathematical language. We also added tiles to

allow the composition of sentences revealing the most common errors.



	A	B	C	D	E	F
1	11	12	13	14	=	
2	56	57	58	59		
3						
4						
5						
6						
7						
8						

Figure 2 - GeoGebra spreadsheet [Image text translation: "to insert a formula, you have to start with an = sign, as shown in the example below"].

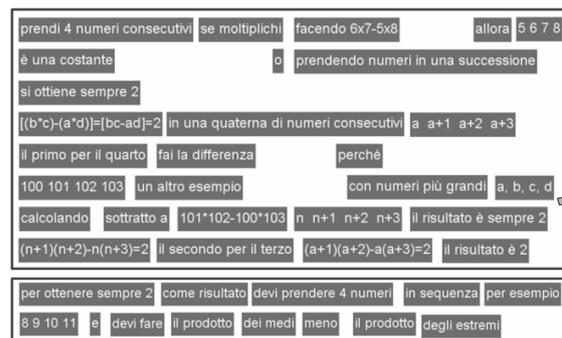


Figure 3 - Interactive Semi-open Question (ISQ) [Image text translation: "to always get 2 - as the result - take 4 numbers - in a row - for instance 8 9 10 11 - and - compute - the product - of the intermediate terms - minus - the product - of the extreme terms"].

6. Sample from experimentation

In this work, we show the significant aspects of an experimentation involving three teachers and around 60 students from three different classes from first and second year of high school. The teachers and the respective classes are from three different places.

During the experiments, the teacher's training interventions and their typologies, the management problems and the involvement/participation of the students and their difficulties were investigated. The first results show that the immersive aspect of storytelling, together with collaborative work and online interactions, typical of platform activities, can lead to a change in students' attitudes towards mathematics and their relationship with the teacher.

In this paper we want to give a taste of what happened, focusing on the Conjecture phase. The Figure 4 shows the starting point of the problematic situation to be faced

by the students. They received some strings of symbols, reported by Federico on a piece of paper, and they tried to understand them.

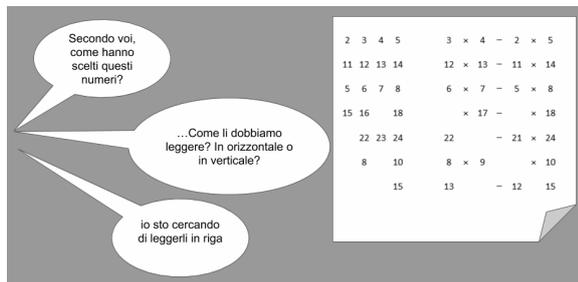


Figure 4 - The problematic situation [Image text translation: “How do you think they chose the numbers?” “Should we read them horizontally or vertically?” “I’m trying to consider them line by line”].

Each line of the paper shows a sequence of four consecutive numbers, followed by their combination under suitable operations. Some lines lack some information (see white spaces). This want to focus the students’ attention to two facts: the consecutiveness of the given number, and the order of the operations with respect to the number sequence.

At beginning the students explore the problematic situation, looking at the data on the sheet and maybe making some trials with new number sequences. The exploration should bring them to make a conjecture, at beginning in form of a verbal statement.

The following scene 1C shows a debate among peers, while they are conjecturing.

Excerpt 1 - Scene 1C:

- Pest: *therefore, in a series of consecutive numbers, if we make the difference between the product of the first and second terms which are always consecutive and between the first and the fourth term which have 2 as difference, the result will always be 2*
- Pest: *If it were done in another way would not be 2*
- Boss: *Yes, for me the reasoning is really okay.*

We can note the emergence of the correct conjecture, although it is supported by a weak argument: “*If it were done in another way would not be 2*”, which has been approved as a very correct reasoning from a peer. It is worthwhile to see that the collaborative mood during the peers’ interaction is supported by the consistent role played by Pest and Boss.

The following scene 2C shows how sometimes there is the need of some personal reflection.

Excerpt 2 - Scene 2C:

- Pest: *I still don’t have clear ideas*
- Boss: *guys! read [the numbers] vertically...!!!!*
- Blogger: *I have read it, but I’m still trying to understand*
- Promoter: *do you have any idea?*

- Boss: *do you realize that going little by little they are numbers that follow one another*
- Boss: *do we read and find ourselves in chat in 5 minutes?*

Promoter: *I have few ideas and confused too*

Promoter: *listen to me for a moment*

Pest: *speak*

Promoter: *2 3 4 5*

Promoter: *the 3 X 4*

Promoter: *2 X 5*

Blogger: *it is 2*

Blogger: *the first line of the second column*

Promoter: *that is the two extremes and the two middle ones*

Promoter: *they all do the same*

Promoter: *there are four terms and the two middle ones, and the two extremes are multiplied*

Promoter: *and then they are subtracted*

Note that all the students are engaged in producing the conjecture. Moreover, we can observe how the intervention of the Boss suggesting, in a deadlock moment, to read again and think alone for five minutes and then come back in the chat, produces the effect that the Promoter, who was the one with “*few ideas and confused too*” now is who successes in understanding which is the formula corresponding to the given sequences.

Let us conclude with the last scene showing how the students become embedded in the narration

Excerpt 3 - scene 3C:

- Pest: *the various results that we derive from the various calculations could not be geographical coordinates?*
- Boss: *after having reasoned with my group, we decided that aliens maybe want to tell us that maybe they are two and this explains why from the quadruplets always two comes out.*

In the following we show an excerpt from the Forum, where the students move to communicate with Gianmaria (that is the expert). After exploring and comparing with peers, once they reach a common conjecture, they propose their statement to Gianmaria (Figure 5).

The verbal statement produced by the students is correct. It is worthwhile to note that in interacting with the “expert”, the production of a symbolic formulation and of an example immediately arise. The crucial point is that the symbolic expression uses the letter a,b,c,d (that refers implicitly to the consecutiveness of the numbers) is not pertinent in order to produce the proof of the conjecture. Note that the “=” in the expression “[bc-ad]=2” is imposed by the students, differently from the previous one which is correct from mathematical point of view. This is a very important issue from the didactical point of view. That is why we foresee an episode of the story dedicated to move from the verbal

conjecture to a symbolic one which is suitable to be manipulated according to mathematical procedures in order to prove the given statement. Figure 6 shows a comics strip to go further to the next episode.

Risposta "Perché viene sempre 2" Gruppo "È vero sì o no?"
 di katiann Tlilgher - lunedì, 16 aprile 2018, 23:26
 Dati 4 numeri consecutivi, in ordine crescente o decrescente, la differenza tra il prodotto del 2° numero per il 3° numero e il prodotto del 1° numero per il 4° numero è sempre uguale a 2.
 Espresso in lettere: dati 4 numeri consecutivi a - b-c-d [(b*c)-(a*d)]=[bc-ad]=2
 Esempio: [(66*67)-(65*68)]=[4422-4420]=2

Figure 5 - An excerpt from the Forum [Image text translation:

Given four consecutive numbers, in ascending or descending order, the difference between the product of the 2nd number and the 3rd number and the product of the 1st number and the 4th number is always equal to 2.

In symbols given four consecutive numbers

$$a-b-c-d [(b*c)-(a*d)] = [bc-ad] = 2$$

$$\text{Example: } [(66*67)-(65*68)] = [4422-4420] = 2 .$$



Figure 6 - A comics strip [Image text translation:

“Yes, good idea. We can say that now we understand the secret of the quadruples” - “But how do we tell them? They probably don't speak our language”].

7. Discussions and Conclusions

As the experimentations made and in progress, the narrative framework, enriched with digital semiotic mediators, seems to engage students both from the motivational point of view and mathematical one, fostering the production of conjectures and supporting them in proving them. The implementation of the different roles in the articulated collaborative tasks, the use of chats and forums, has contributed to let the students experience and, perhaps, internalize a way of being a mathematician.

During the experimentations, the students asked for more digital tiles to better formalize their conjectures. Some digital tiles were added in real time by the researcher in charge of this activity. The issue has been finally overcome modifying the ISQ to allow students to create their own digital tiles. This feature will be tested in future trails.

We started the project with the following research question in mind: “how to support the development of students’ argumentative capabilities in mathematics” by means of involving and inclusive activities. We thought (and found) that technology is certainly part of the answer. In fact, we believe that rich online teaching and learning environments (such as Moodle and GeoGebra) should no longer be considered in merely

instrumental terms, but as enabling technologies, without which much advanced teaching, like the one described in this work, is almost impossible to achieve. Technology can play an important role as a semiotic mediator using artifacts where mathematical knowledge is embedded and ready to be explored by students. A DIST-M can be implemented in the teaching daily practice choosing suitable problems (i.e. well integrated within the classroom curriculum) and foreseeing a final assessment. To this purpose, we are currently working on a new (evaluation) phase at the end of the story. It is also worth remarking that, after a few runs, the role should be interiorized enough to relax some of technological requirements for the activity to work.

We conclude underlying the fundamental insight that ICT can offer to the teacher. By observing communications and spontaneous connections between students the teacher can timely detect dangerous misconceptions and classroom dynamics and act consequently. In fact, they provide a sort of augmented reality whose importance will grow with the arrival of the forthcoming Learning Analytics tools.

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